Revision Notes for Core 4

Differentiating Trigonometric Functions

$\frac{d}{dx}$ sin x = cos x and $\frac{d}{dx}$ cos x = - sin x. All the rest can be found by expressing the function in terms or sin or cos then using the product, chain or quotient rule.

Use formula booklet to help and look at both differentiating and integrating columns as one is the reverse of the other.

Use 2sinAcosA = sin 2A, $2\cos^2 A = 1 + \cos 2A$ and $2\sin^2 A = 1 - \cos 2A$ to change into expressions you can differentiate or integrate. e.g. from $\cos^2 x$ into $\frac{1}{2} + \frac{1}{2}\cos 2A$

Remember $\frac{d}{dx}$ sin 2x = 2 cos 2x (chain rule) and $\int \sin 2x = -\frac{1}{2}$ cos 2x + c

Integration

Either by parts or substitution

By parts: $\int u\frac{dv}{dx}dx = uv - \int \frac{du}{dx}v\,dx$ which means call one bit u and the other $\frac{dv}{dx}$ so integrate the $\frac{dv}{dx}$ to give v and differentiate the u to give $\frac{du}{dx}$ then sub in to the formula.

Remember: Diff. one and Int. the other and choose to diff. the one that will disappear.

If still have a product in the second integral bit, swap around which bit to integrate and do again then if still horrid, do substitution.

Substitution

Always given the substitution so differentiate this then substitute in for dx and then can cancel OR rearrange and express x in terms of u then differentiate and substitute in.

Example $\int \frac{1}{1-x^{1/2}}$ dx where $u = 1 - x^{1/2}$

$x = (1 - u)^2$ and $\frac{dx}{du} = -2(1 - u) = 2u - 2$ so dx = (2u – 2)du

$\int \frac{1}{u}(2u - 2)\,du = \int (2 - \frac{2}{u})\,du = 2u - 2\ln|u| + c$

Substitute back in for u therefore $= 2(1 - x^{1/2}) - 2\ln|1 - x^{1/2}| + c$

Parametric Equations

Uses an equation for x and y separately but with an extra variable **t** to link the two together. To change to a cartesian equation, solve simultaneously to find the value of t then write as y=mx + c.

If the parametric equations are for curves, then use chain rule to find the gradient at a particular point, e.g. $\frac{dy}{dx} = \frac{dt}{dx} \times \frac{dy}{dt}$

e.g. if $y = t^3$ and x = 2t then find $\frac{dy}{dt}$ and $\frac{dx}{dt}$ then invert $\frac{dx}{dt}$ and multiply the two together.

Remember to use the 2 equations to find the x and y coordinates then use y = mx +c if finding the equation of a tangent or normal.

Binomial Expansion

$(1 + x)^n = 1 + \frac{n}{1}x + \frac{n(n-1)}{1x2}x^2 + \frac{n(n-1)(n-2)}{1x2x3}x^3 + \ldots$ **THIS IS IN THE FORMULA BOOKLET**

If $(1 + mx)^n$ then remember that it is $(mx)^2$ not just mx^2 and you may need to take out a common factor to get it in the binomial form. Also used when expressing partial fractions and expanding them.

Rational Functions and Partial Fractions

To do $\frac{a}{b} \times \frac{c}{d}$ factorise anything that you can then cancel down.

To do $\frac{a}{b} \div \frac{c}{d}$ turn second fraction upside down then multiply.

To do $\frac{a}{b} + \frac{c}{d}$ make one fraction by cross multiplying so $\frac{ad+bc}{bd}$ or make a common denominator then combine.

To do the reverse (make two separate fractions from one), factorise the denominator into two factors, express as two fractions each with one of the factors as the denominator and A and B as the numerators, cross multiply then solve for A and B.

e.g. $\frac{7x-8}{(2x-1)(x-2)} = \frac{A}{2x-1} + \frac{B}{x-2}$ then $\frac{7x-8}{(2x-1)(x-2)} = \frac{A(x-2)+B(2x-1)}{(2x-1)(x-2)}$ then set $x=\frac{1}{2}$ to find A and x=2 to find B.

If there is a repeated root (e.g. denominator may be $(2x+3)(x-2)^2$ then 3 fractions $\frac{A}{2x+3} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$

Multiply top and bottom by 2x + 3 then A is on its own and B and C have (2x+3) on the top then make x= - 3/2 and this will eliminate B and C to find A. Now do the same for (x – 2) to find B and so on.

Division of Polynomials

Use long division to find the quotient (how many times it goes in) and the remainder then can rearrange and thus integrate fractions with polynomials in them.

$(20x^3 - x^2 - 4x - 7) \div (4x + 3) =$

$5x^2 - 4x + 2$ (quotient)

$4x+3 \,\overline{)\, 20x^3 - x^2 - 4x - 7}$

$\underline{20x^3 + 15x}$

$-16x^2 - 4x - 7$

$\underline{-16x^2 - 12x}$

$8x - 7$

$\underline{8x + 6}$

-13 (remainder)

So $\int \frac{(20x^3 - x^2 - 4x - 7)}{(4x + 3)}\, dx = \int (5x^2 - 4x + 2) - \frac{13}{(4x + 3)}\, dx$ and can integrate the right hand side so find the solution to the left hand side.

Differential Equations

Get the x's and dx together on one side and the y's and dy on the other then integrate either side. This gives you an equation in terms of x and y rather than dx and dy. Then use the information you are given to find the constant of integration then can use the equation to solve the problem. Put the constant of integration on one side only.

Example If $\frac{dy}{dx} = y^2 \sqrt{x}$ then $\frac{1}{y^2}\, dy = \sqrt{x}\, dx$

$\int \frac{1}{y^2}\, dy = \int \sqrt{x}\, dx$ therefore $\frac{-1}{y} = \frac{2}{3} x^{\frac{3}{2}} + c$

Implicit Equations

These are where the x's and y's occur more than once so cannot be rearrange to make x or y the subject.

To find $\frac{dy}{dx}$ of an curve defined implicitly, differentiate with respect to x but the y terms differentiate from f(y) to f'(y) $\frac{dy}{dx}$ using the chain rule. Then rearrange to make $\frac{dy}{dx}$ the subject and this is the gradient formula.

Example If the equation on a curve is $x^2 - 2xy + 4y^2 = 12$, differentiating with respect to x gives you $2x - 2y - 2x\frac{dy}{dx} + 8y\frac{dy}{dx} = 0$ (the +2xy is differentiated using the product rule)

$\frac{dy}{dx}(-2x + 8y) = -2x + 2y$ and so $\frac{dy}{dx} = \frac{-2x+2y}{-2x+8y} = \frac{-x+y}{-x+4y}$ (this is the gradient formula)

For the coordinates of a point where the gradient is 0, then –x + y =0 so rearrange and substitute in to the equation of the curve to find the point that solves them both.

Vectors

Describes a translation either as a column vector $\begin{pmatrix}4\\5\end{pmatrix}$ or a letter e.g **p** or multiples of basic unit vectors e.g. 4**i + 5j** where **i**= $\begin{pmatrix}1\\0\end{pmatrix}$ and **j**=$\begin{pmatrix}0\\1\end{pmatrix}$

To get vectors of directions you aren't given, use the commutative rule which means draw the network and get from A to B using a different route not direct. To show that a point B is on the line AC, show that the vector AB is a multiple of the vector AC.

The vector equation of a line is **r** = **a** + t**p** (**a** is the position vector of a point on the line and **p** is the displacement vector of the line (the direction the line is going in or the movement in the x, y and z direction to get from point A to point B)

Find **p** by subtracting the vectors of 2 points on the line. **a** is the vector of the first point.

Example

If points A and B have coordinates (-5,3,) and (-2,9,) then the vector equation of the line between them is **p** = $\begin{pmatrix}-2\\9\end{pmatrix} - \begin{pmatrix}-5\\3\end{pmatrix} = \begin{pmatrix}3\\6\end{pmatrix}$ so the equation is **r** = $\begin{pmatrix}-5\\3\end{pmatrix} + t\begin{pmatrix}3\\6\end{pmatrix}$

Same process if in 3D.

t is a scalar quantity that tells you how far along the line to move.

You can write this equation using basic unit vectors or column vectors.

To find if and where 2 lines intersect, make one equal to the other and solve for the x,y and z values (or the i,j and k values) to find the size of t.

Do for all 3 coordinates to check that they do cross in all directions with the same value for t. If they do not cross in all directions, the lines are **skew.**

Scalar Product of Vectors

a.b = |a| |b| cos θ where |a| and |b| is the magnitude (length) of the displacement vectors and found using Pythagoras and θ is the angle between the vectors.

a.b means find the product of the x, y and z values and sum them.

$\begin{pmatrix}l\\m\\n\end{pmatrix} \cdot \begin{pmatrix}u\\v\\w\end{pmatrix}$ = lu + mv + nw so combining gives **|a| |b| cos θ = lu + mv + nw**

If 2 vectors are perpendicular, then p.q = 0