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A22 SOLVE LINEAR INEQUALITIES IN ONE **OR TWO** VARIABLE**(S)**, **AND QUADRATIC INEQUALITIES IN ONE VARIABLE;** REPRESENT THE SOLUTION SET ON A NUMBER LINE, **USING SET NOTATION AND ON A GRAPH (higher tier)**

You should be able to solve quadratic equations of the form $ax^2 + bx + c = 0$

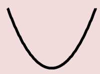
e.g. $x^2 - 3x - 4 = 0$ $(x - 4)(x + 1) = 0$ $x = 4$ or $x = -1$

e.g. $3x^2 - 14x + 8 = 0$ $(3x - 2)(x - 4) = 0$ $x = \frac{2}{3}$ or $x = 4$

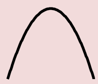
e.g. $x^2 = 10 - 3x$ $x^2 + 3x - 10 = 0$ $(x + 5)(x - 2) = 0$ $x = -5$ or $x = 2$

You should also know the shape of a quadratic curve.

If the coefficient of x^2 is positive, the curve is '**smiling**'.



If the coefficient of x^2 is negative, the curve is '**frowning**'.



If $f(x) > 0$ or $f(x) \geq 0$ we want the values of x where $f(x)$ is **above** the x-axis.

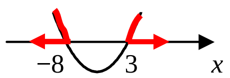
If $f(x) < 0$ or $f(x) \leq 0$ we want the values of x where $f(x)$ is **below** the x-axis.

EXAMPLE 1

Solve $x^2 + 5x - 24 \geq 0$

$(x + 8)(x - 3) \geq 0$ ← First factorise your quadratic expression

Critical values are $x = -8$ and $x = 3$ ← Solve $(x + 8)(x - 3) = 0$



← Always draw a sketch of your curve
Show where the curve cuts the x-axis
by solving $(x + 8)(x - 3) = 0$

$x \leq -8$ and $x \geq 3$ ← We want the area where $y \geq 0$

If you are asked to write the **solution set** of the inequality $x^2 + 5x - 24 \geq 0$ then the answer would be: $\{x : x \leq -8,\ x \geq 3\}$

NOTE: There are TWO regions so we write the answer as TWO inequalities.

EXAMPLE 2

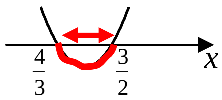
Find the solution set of the inequality $6(x^2 + 2) < 17x$

$6x^2 + 12 < 17x$ ← First expand the bracket

$6x^2 - 17x + 12 < 0$ ← Rearrange to the form $ax^2 + bx + c < 0$

$(3x - 4)(2x - 3) < 0$ ← Factorise in order to find where it cuts the x-axis

Critical values are $x = 4/3$ and $3/2$ ← Solve $(3x - 4)(2x - 3) = 0$



← Sketch the curve and shade below the axis

$\frac{4}{3} < x < \frac{3}{2}$ ← We want the region where f(x) is **below** the x-axis

Solution set = $\{ x : \frac{4}{3} < x < \frac{3}{2} \}$ ← Make sure your answer is given in the correct form

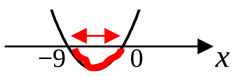
NOTE: There is ONE region so we write the answer as ONE inequality.

EXAMPLE 3

Solve $x(x + 9) \leq 0$

$x(x + 9) \leq 0$ ← This is already factorised with 0 on one side so there is no need to expand the brackets

Critical values are $x = 0$ and $x = -9$



← Sketch the curve and shade below the axis

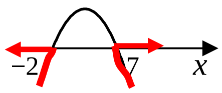
$-9 < x < 0$ ← We want the region where f(x) is **below** the x-axis. There is only one region so write as one inequality

EXAMPLE 4

Solve the inequality $14 + 5x < x^2$

$14 + 5x - x^2 < 0$ ← Rearrange to the form $ax^2 + bx + c < 0$

$(2 + x)(7 - x) < 0$ ← Factorise in order to find where it cuts the x-axis



← The curve is **'frowning'** as we have $-x^2$

$x < -2$ and $x > 7$ ← We want the region where f(x) is **below** the x-axis

OR

$14 + 5x - x^2 < 0$ ← Rearrange to the form $ax^2 + bx + c < 0$

$x^2 - 5x - 14 > 0$ ← Multiply each term by −1 which changes $<$ to $>$

$(x + 2)(x - 7) > 0$ ← Factorise in order to find where it cuts the x-axis

← The curve is **'smiling'** as we have $+x^2$

$x < -2$ and $x > 7$ ← This method gives the same answer as the 1st method

EXERCISE:

1. Solve each of these inequalities.

(a) $x^2 + 9x + 18 \leq 0$
(b) $x^2 - x - 20 < 0$
(c) $(x - 2)(x + 7) > 0$
(d) $x^2 - 5x \geq 0$
(e) $2x^2 - 11x + 12 < 0$
(f) $(5 + x)(1 - 2x) \geq 0$
(g) $15 + 2x - x^2 \leq 0$
(h) $21 - x - 2x^2 > 0$
(i) $x(5x - 2) > 0$
(j) $x^2 - 2x > 35$

2. Find the solution set for each of these inequalities.

(a) $x^2 - 4x + 3 \leq 0$
(b) $x^2 + x - 42 < 0$
(c) $x(x + 2) > 48$
(d) $3x^2 + 14x - 5 \geq 0$
(e) $2x^2 > 11x - 12$
(f) $16 - x^2 \leq 6x$
(g) $7 + 2(4x^2 - 15x) \leq 0$
(h) $x^2 - 4(x + 6) > 8$
(i) $3x(5 - x) > 0$
(j) $(x + 5)^2 \geq 1$

3. Solve $\dfrac{x^2 + 12}{2} \geq 4x$

4. Find the solution set for which $15 + 2x \leq x^2$

5. Find the set of values for which $6 + x \geq x^2$ and $x + 2 < x^2$

6. Find the solution set for $(x - 3)(2x + 3) < 2x(1 - 2x) - 5$

ANSWERS:

1. (a) $-6 \le x \le -3$ (b) $-4 < x < 5$
(c) $x < -7$ or $x > 2$ (d) $x \le 0$ or $x \ge 5$
(e) $\frac{3}{2} < x < 4$ (f) $-5 \le x \le \frac{1}{2}$
(g) $x \le -3$ or $x \ge 5$ (h) $-\frac{7}{2} < x < 3$
(i) $x < 0$ or $x > \frac{2}{5}$ (j) $x < -5$ or $x > 7$

2. (a) $\{ x : 1 \le x \le 3 \}$ (b) $\{ x : -7 < x < 6 \}$
(c) $\{ x : x < -8, x > 6 \}$ (d) $\{ x : x \le -5, x \ge \frac{1}{3} \}$
(e) $\{ x : x < \frac{3}{2}, x > 4 \}$ (f) $\{ x : x \le -8, x \ge 2 \}$
(g) $\{ x : \frac{1}{4} \le x \le \frac{7}{2} \}$ (h) $\{ x : x < -4, x > 8 \}$
(i) $\{ x : 0 < x < 5 \}$ (j) $\{ x : x \le -6, x \ge -4 \}$

3. $x \le 2$ or $x \ge 6$

4. $\{ x : x < -1, x > 2 \}$

5. $-2 \le x < -1$ and $2 < x \le 3$

6. $\{ x : -\frac{1}{2} < x < \frac{4}{3} \}$