

A4 SIMPLIFY AND MANIPULATE ALGEBRAIC EXPRESSIONS (INCLUDING THOSE INVOLVING SURDS AND ALGEBRAIC FRACTIONS) BY EXPANDING PRODUCTS OF TWO OR MORE BINOMIALS (higher tier)

You should already be able to expand algebraic expressions of the form $(ax + b)(cx + d)$.

e.g. $(2x + 1)(3x - 2) = 6x^2 - 4x + 3x - 2 = 6x^2 - x - 2$

e.g. $(5x + 4)(5x - 4) = 25x^2 - 20x + 20x - 16 = 25x^2 - 16$

We are now going to algebraic expressions of the form $(ax + b)(cx + d)(ex + f)$.

EXAMPLE 1

Expand and simplify $(x - 2)(2x + 3)(x + 7)$

$$(x - 2)(2x + 3) = 2x^2 + 3x - 4x - 6$$

← First expand two of the brackets
(You may prefer to use the grid method)

$$= 2x^2 - x - 6$$

← Simplify

Now $(x - 2)(2x + 3)(x + 7) = (x + 7)(2x^2 - x - 6)$

$$= x(2x^2 - x - 6) + 7(2x^2 - x - 6)$$

← Multiply your expansion by each term in the 3rd bracket

$$= 2x^3 - x^2 - 6x + 14x^2 - 7x - 42$$

$$= 2x^3 + 13x^2 - 13x - 42$$

← Simplify

EXAMPLE 2

Show that $(2x + 5)(x - 1)(4x - 3) = 8x^3 + 6x^2 - 29x + 15$ for all values of x .

$$(2x + 5)(x - 1) = 2x^2 - 2x + 5x - 5$$

← First expand any two of the brackets.

$$= 2x^2 + 3x - 5$$

← Simplify

Now $(2x + 5)(x - 1)(4x - 3) = (4x - 3)(2x^2 + 3x - 5)$

$$= 4x(2x^2 + 3x - 5) - 3(2x^2 + 3x - 5)$$

← Multiply your expansion by each term in the 3rd bracket

$$= 8x^3 + 12x^2 - 20x - 6x^2 - 9x + 15$$

← Remember the minus outside the 2nd bracket changes each sign inside the 2nd bracket

$$= 8x^3 + 6x^2 - 29x + 15$$

← Simplify

To simplify the product of three binomials, first expand any two of the brackets and then multiply this answer by each term in the third bracket.

EXAMPLE 3

Expand and simplify $(x + 3)(x - 3)(2x + 1)(5x - 6)$

$$(x + 3)(x - 3) = x^2 - 9$$

← Expand two of the brackets

$$(2x + 1)(5x - 6) = 10x^2 - 7x - 6$$

← Expand the other two brackets

$$(x + 3)(x - 3)(2x + 1)(5x - 6)$$

$$= (x^2 - 9)(10x^2 - 7x - 6)$$

← Use the two expansions above

$$= x^2(10x^2 - 7x - 6) - 9(10x^2 - 7x - 6)$$

← Multiply the 2nd bracket by each term in the 1st bracket

$$= 10x^4 - 7x^3 - 20x^2 - 90x^2 + 63x + 54$$

$$= 10x^4 - 7x^3 - 110x^2 + 63x + 54$$

← Simplify

To simplify the product of four binomials, first expand any two of the brackets and then expand the other two brackets and then multiply these answers.

EXERCISE:

1. Expand and simplify

(a) $(x + 1)(x + 4)(x + 5)$

(b) $(2x + 7)(3x + 1)(x + 8)$

(c) $(x - 3)(x - 1)(2x - 3)$

(d) $(3x + 8)(x - 2)(2x - 5)$

(e) $(5x - 1)(2x + 5)(3x - 2)$

(f) $(4x + 1)(2x + 7)(4x - 1)$

(g) $(x + 4)^2(3x - 7)$

(h) $(6x + 5)(2x - 1)^2$

(i) $(x - 1)(x + 1)(4x - 1)(2x - 5)$

(j) $(x + 5)^2(x - 2)^2$

2. Show that $(2x + 3)^3 = 8x^3 + 36x^2 + 54x + 27$ for all values of x .

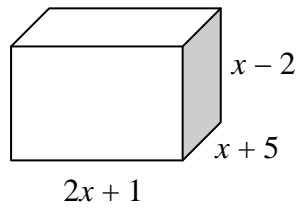
3. Show that $(x - 4)^2(x + 3)$ simplifies to $x^3 + ax^2 + bx + c$ where a , b and c are integers.

4. Express $(3x - 1)^4$ in the form $ax^4 + bx^3 + cx^2 + dx + e$ where a , b , c , d and e are integers.

5. $(3x + 5)(x - 4)(3x - 2) = 9x^3 + Ax^2 + Bx + 40$
Work out the value of A and the value of B .

6. $(x - 3)(2x + 1)(Ax + 1) = 8x^3 + Bx^2 + Cx - 3$
Work out the value of A , the value of B and the value of C .

7. Here is a cuboid.



All measurements are in centimetres.

Show that the volume of the cuboid is $(2x^3 + 7x^2 - 17x - 10) \text{ cm}^3$.

8. $f(x) = 3x^3 - 2x^2 + 4$

Express $f(x + 2)$ in the form $ax^3 + bx^2 + cx + d$.

9. The smallest of three consecutive positive odd numbers is $(2x - 1)$.

Work out the product of the three numbers.

Give your answer in the form $ax^3 + bx^2 + cx + d$.

ANSWERS:

1. (a) $x^3 + 10x^2 + 29x + 20$
(b) $6x^3 + 71x^2 + 191x + 56$
(c) $2x^3 - 11x^2 + 18x - 9$
(d) $6x^3 - 11x^2 - 42x + 80$
(e) $30x^3 + 49x^2 - 61x + 10$
(f) $32x^3 + 112x^2 - 2x - 7$
(g) $3x^3 + 17x^2 - 8x - 112$
(h) $24x^3 - 4x^2 - 14x + 5$
(i) $8x^4 - 22x^3 - 3x^2 + 22x - 5$
(j) $x^4 + 6x^3 - 11x^2 - 60x + 100$
2. Proof
3. $x^3 - 5x^2 - 8x + 48$
4. $81x^4 - 108x^3 + 54x^2 - 12x + 1$
5. $A = -27$ $B = -46$
6. $A = 4$ $B = -18$ $C = -17$.
7. Proof
8. $x^3 + 4x^2 + 4x + 12$
9. $8x^3 - 12x^2 - 2x - 3$