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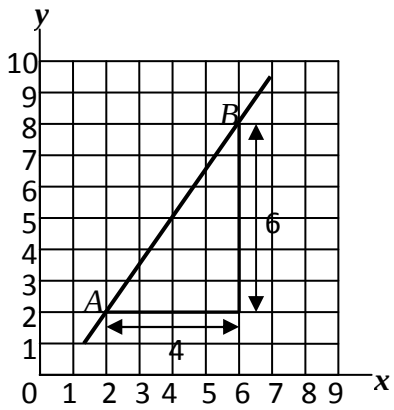
R14 INTERPRET THE GRADIENT OF A STRAIGHT LINE GRAPH AS A RATE OF CHANGE; RECOGNISE AND INTERPRET GRAPHS THAT ILLUSTRATE DIRECT AND INVERSE PROPORTION (foundation and higher tier)

You should know how to find the gradient of a straight line from a diagram or graph.
This next section is just for revision.

REVISION:
The slope of a line is called its gradient.
The larger the value of the gradient, the steeper the slope.

Calculating the gradient

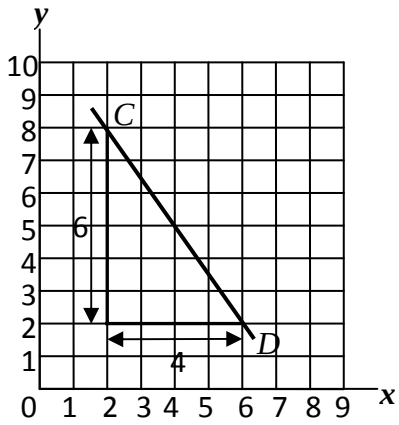
The gradient of a straight line can be calculated by drawing a right-angled triangle between **any** two points lying on the line.



In the above graph a right-angled triangle is constructed between points *A* and *B* (lying on the line). The gradient of the line passing through the points *A* and *B* is given by

$$\text{gradient} = \frac{\text{change in } y}{\text{change in } x} = \frac{6}{4} = \frac{3}{2}$$

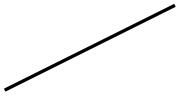
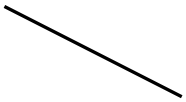
If the line is sloping down then a negative sign is placed in front of the answer.



The gradient of the line passing through the points C and D is given by

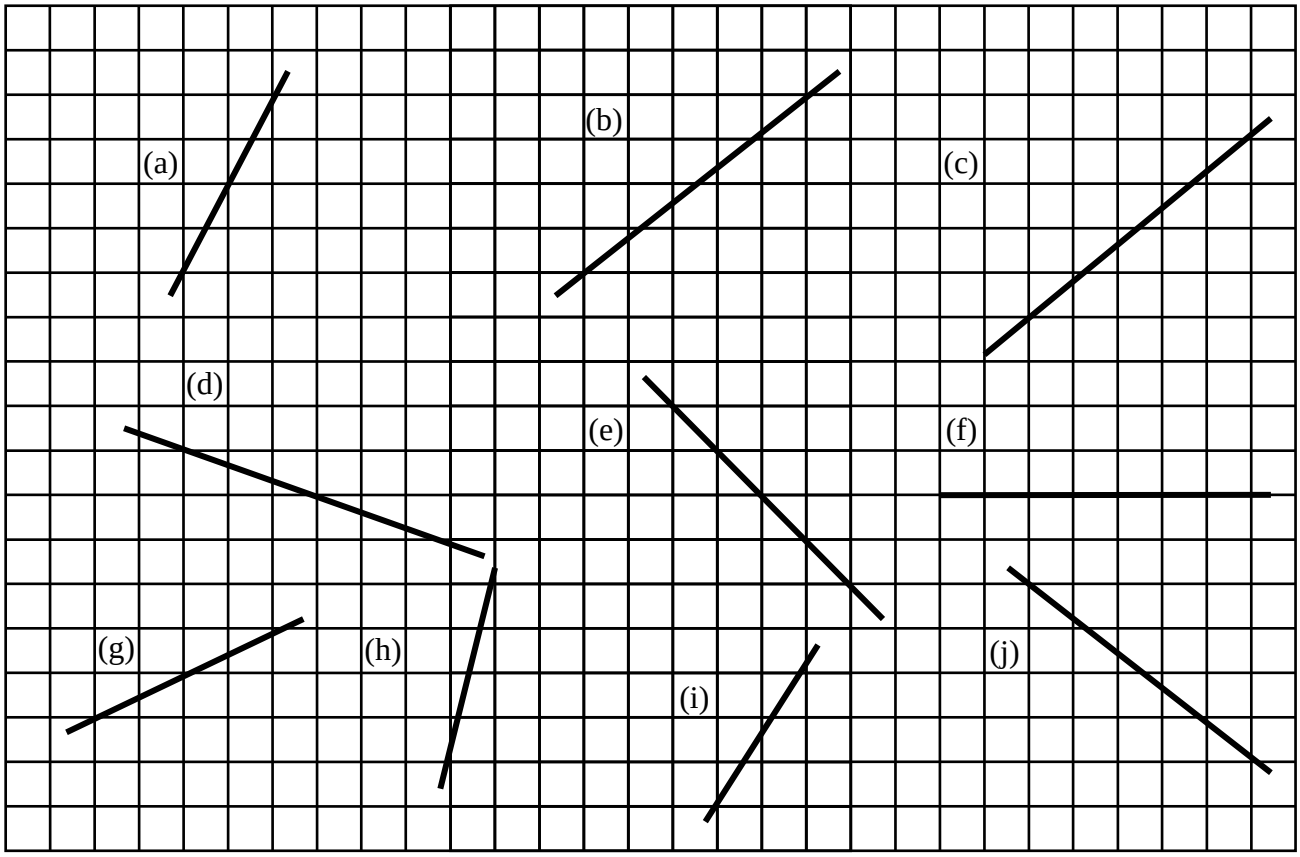
$$\text{gradient} = \frac{\text{change in } y}{\text{change in } x} = -\frac{6}{4} = -\frac{3}{2}$$

NOTE:

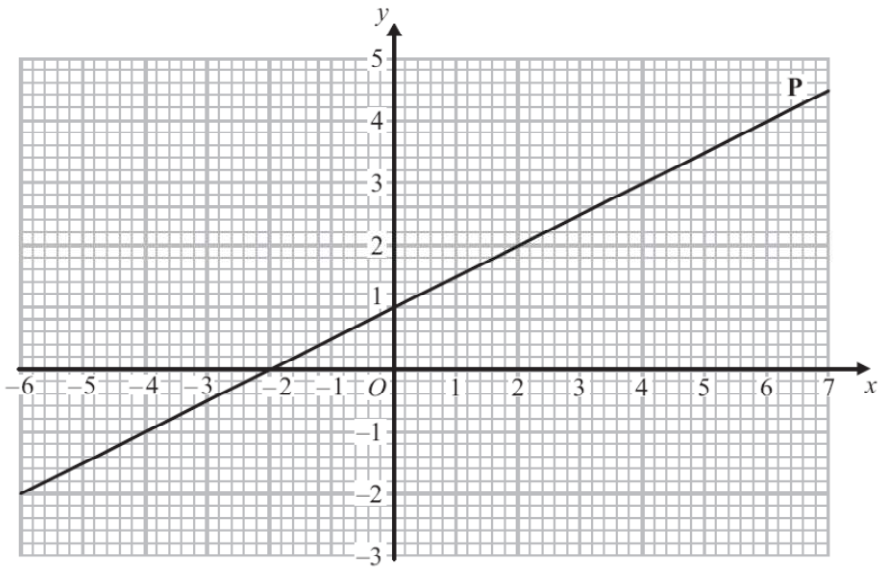
Shape	Sign of gradient
	positive
	negative

REVISION EXERCISE:

1. Find the gradient of each line below, leaving your answer as a fraction in its simplest form.



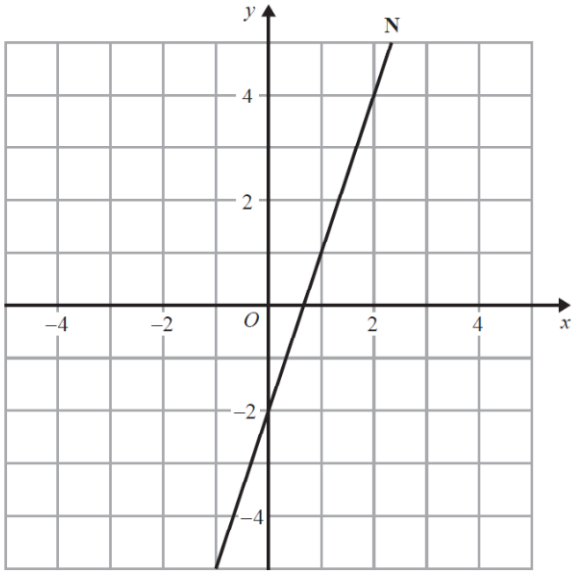
2. The straight line **P** has been drawn on a grid.



Find the gradient of the line **P**.

3. The points (1, 4) and (4, −2) lie on the straight line **L**.
Work out the gradient of **L**.

4. The line **N** is drawn below.



Find the gradient of the straight line N.

5.

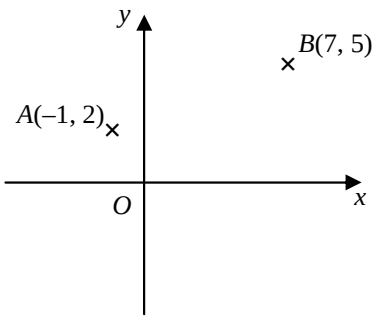


Diagram NOT accurately drawn

(a) Work out the gradient of the line *AB*.

P is the point (–4, 4) and *Q* is the point (1, –5)

(b) Find the gradient of *PQ*.

INTERPRETING GRADIENTS OF STRAIGHT LINES

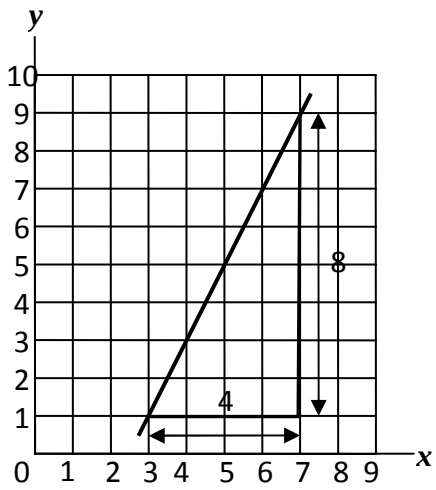
Rates of change

> The **rate of change** of a line is a measure of how steep it is.
> In mathematics we also call this the gradient.

The rate of change is a single value that describes the gradient of a line:

> If the rate of change is increasing it has a positive gradient
> If the rate of change is decreasing it has a negative gradient

Consider,



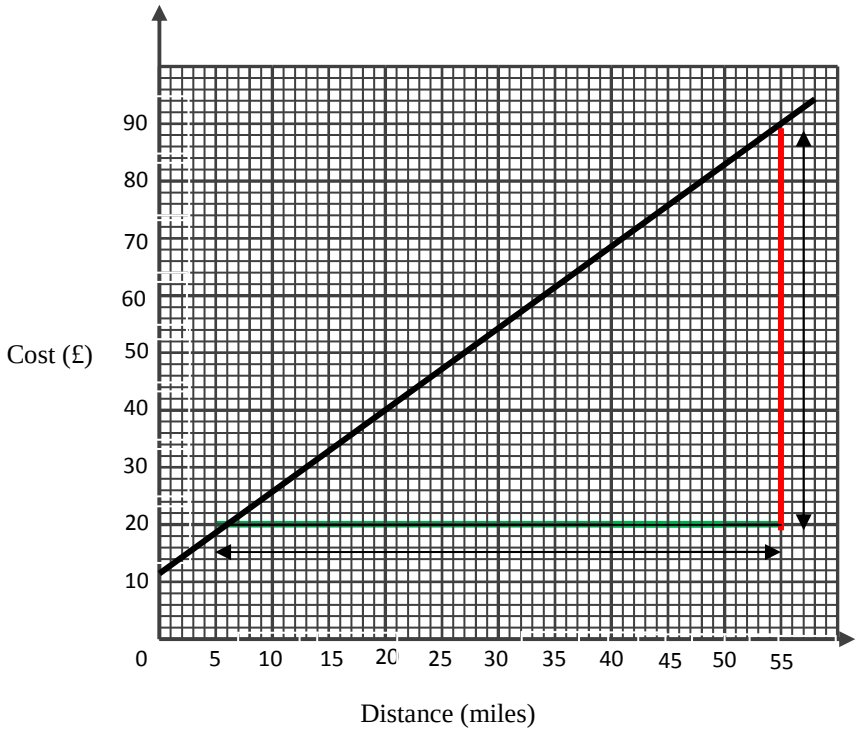
$$\text{gradient} = \frac{\text{change in } y}{\text{change in } x} = \frac{8}{4} = \frac{2}{1}$$

So the rate of change is 2

The interpretation would be for every 1 unit increase in x there is 2 units increase in y

EXAMPLE 1

Bill uses his van to deliver parcels.
The graph can be used to find the total cost of having a parcel delivered by Bill.



(a) Work out the gradient of the line.

(b) Interpret the gradient

Always draw a triangle (shown in **red** and **green** on the diagram).
Try to draw the triangle so that it goes through easy points to read on the graph.

(a)

Change in cost (y) (in red) $= 90 - 20 = 70$ ← Read the y-coordinates on your triangle

Change in distance (x) (in green) $= 55 - 5 = 50$ ← Read the x-coordinates on your triangle

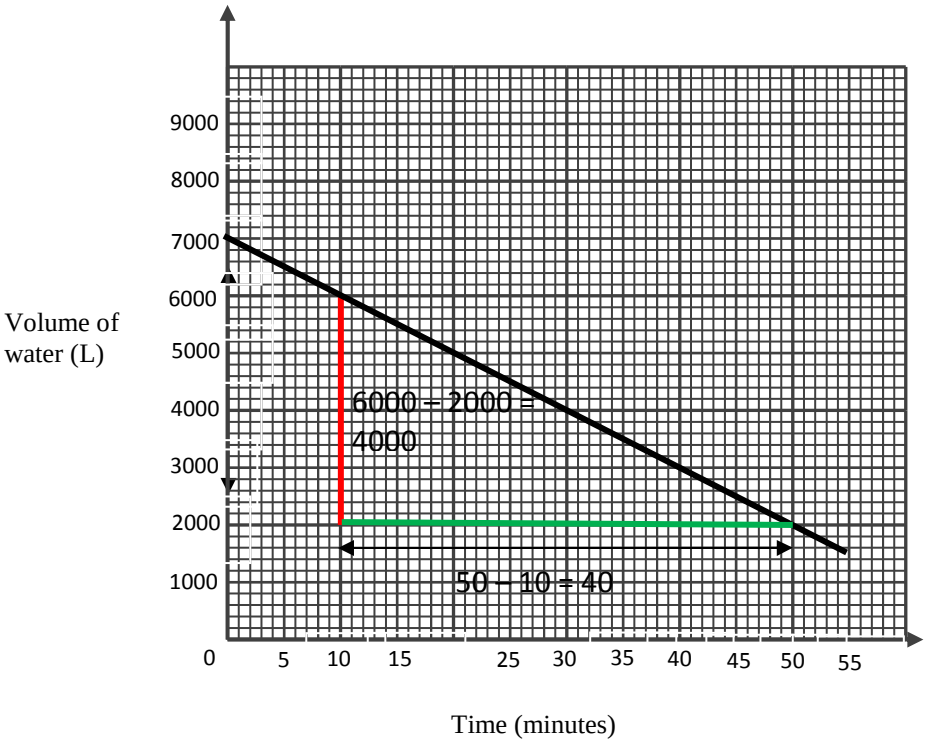
$$\text{gradient} = \frac{\text{change in cost}}{\text{change in distance}} = \frac{70}{50} = 1.4$$

(b) For every 1 mile travelled the cost of having a parcel delivered increases by £1.40

← 1.4 = 1.4 ÷ 1 so can find increase in y (cost in £) for each 1 unit in x (distance in miles)

EXAMPLE 2

A tank has been emptied and the volume of water was recorded.



(a) Work out the gradient of the line.

(b) Interpret the gradient

Always draw a triangle (shown in **red** and **green** on the diagram).
Try to draw the triangle so that it goes through easy points to read on the graph.

(a)

Change in volume (y) (in red) = 6000 − 2000 = 4000 ← Read the y-coordinates on your triangle

Change in time (y) (in green) = 50 − 10 = 40 ← Read the x-coordinates on your triangle

$$\text{gradient} = \frac{\text{change in volume}}{\text{change in time}} = -\frac{4000}{40} = -100$$

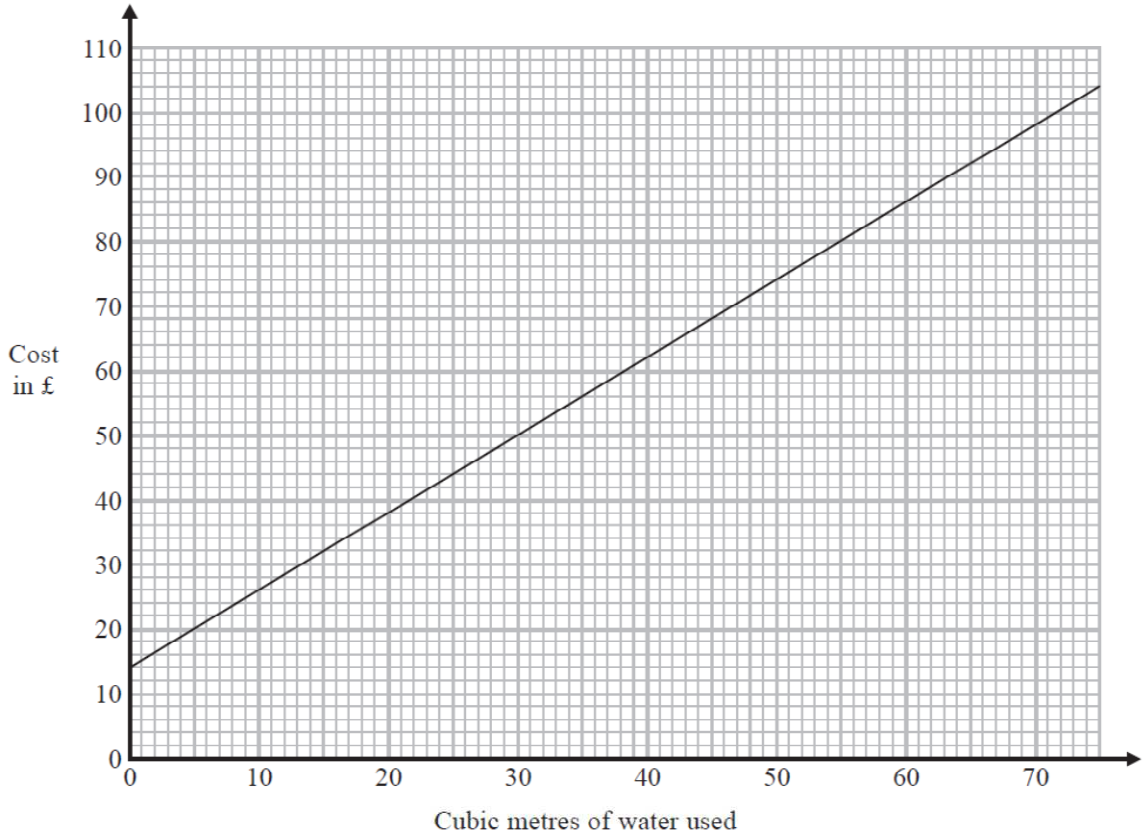
The line slopes down so the gradient is negative

(b) For every 1 minute the tank empties 100 litres ← The negative gradient shows that the tank is emptying.

EXERCISE 1:

1. A water company charges customers a fixed standing charge plus an additional cost for the amount of water, in cubic metres, used.

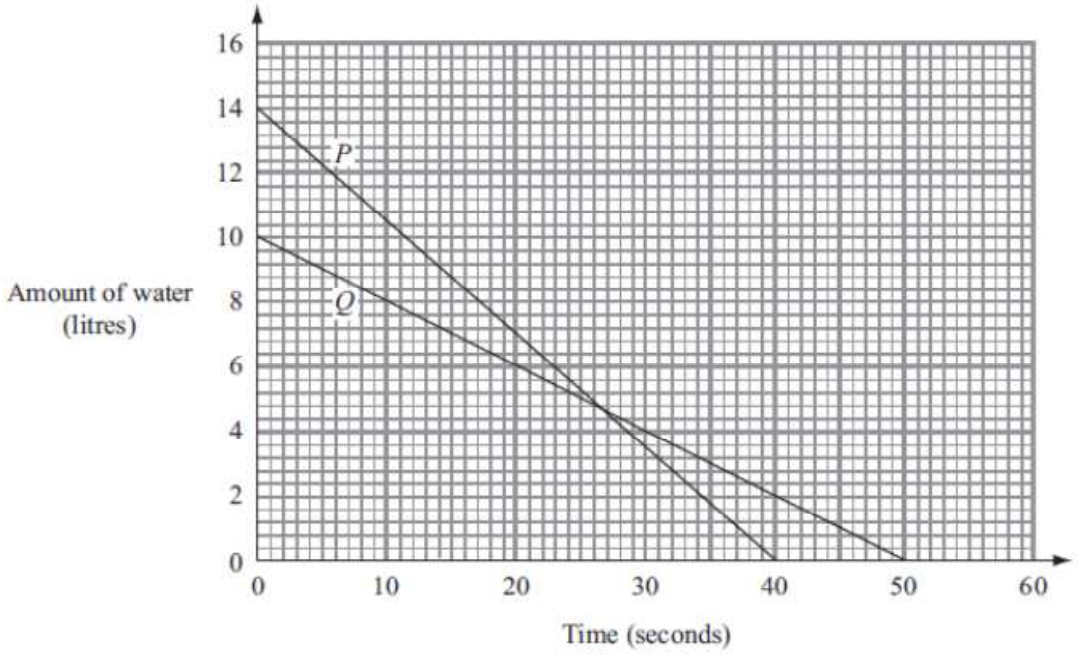
The graph shows information about the total cost charged.



(a) Write down the fixed standing charge.
(b) Work out the gradient of the line.
(c) Interpret the gradient.

2. Water is leaking out of two containers.
The water started to leak out of the containers at the same time.

The straight line *P* shows information about the amount of water, in litres, in container *P*.
The straight line *Q* shows information about the amount of water, in litres, in container *Q*.



(a)(i) Work out the gradient of line *P*.

(ii) Interpret the gradient

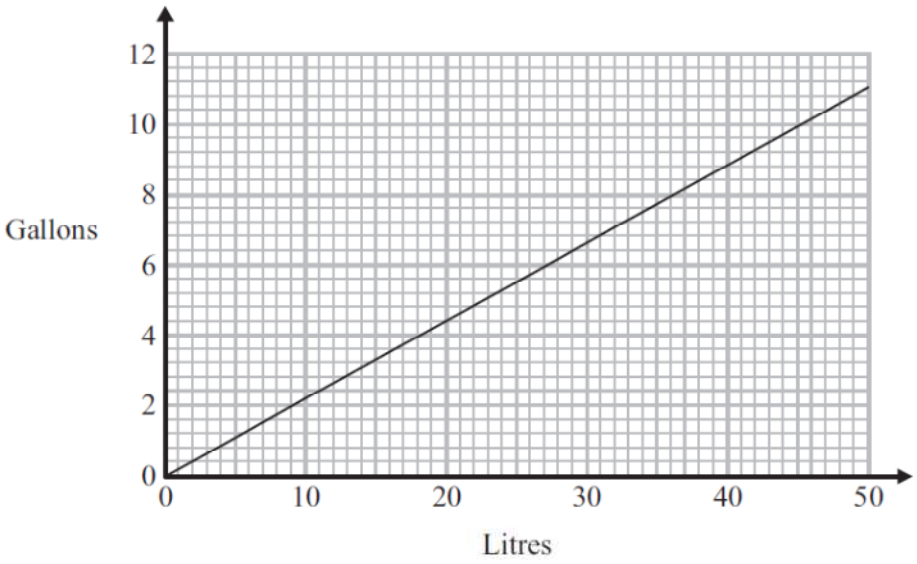
(b)(i) Work out the gradient of line *Q*.

(ii) Interpret the gradient

One container will become empty first.

(c) Which container?
You must explain your answer.

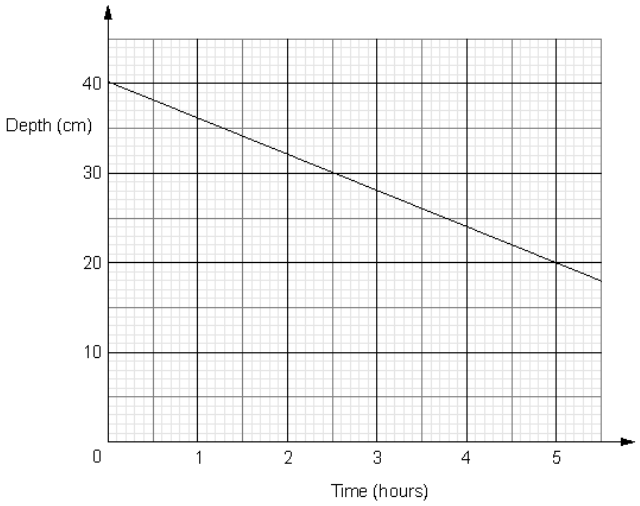
3. You can use this graph to convert between litres and gallons.



(a) Work out the gradient of line *P*.

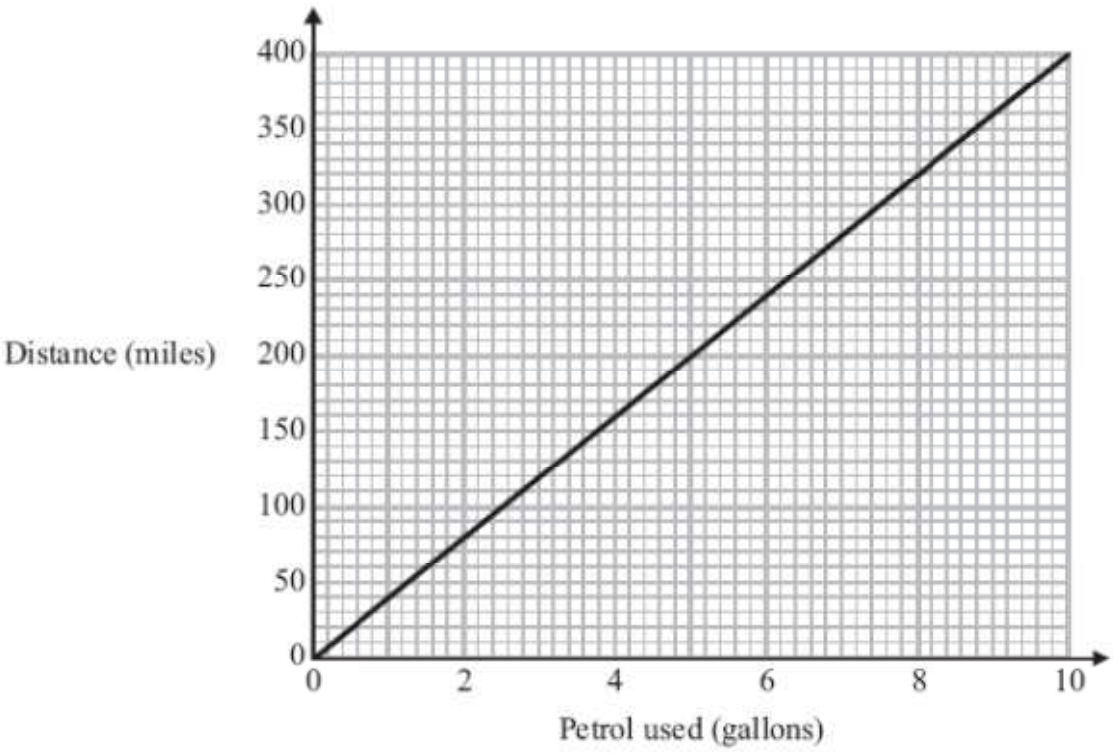
(b) Comment on the value of the gradient.

4. Water flows out of a cylindrical tank at a constant rate.
The graph shows how the depth of water in the tank varies with time.



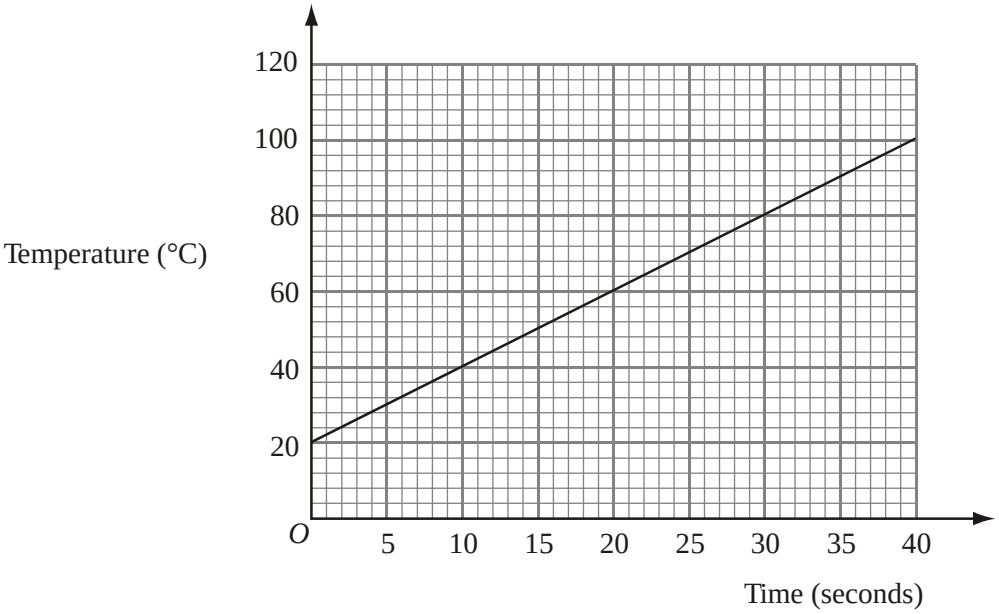
(a) Work out the gradient of the straight line.
(b) Give a practical interpretation of the value you worked out in part (a).

5. The graph shows information about the distances travelled by a car for different amounts of petrol used.



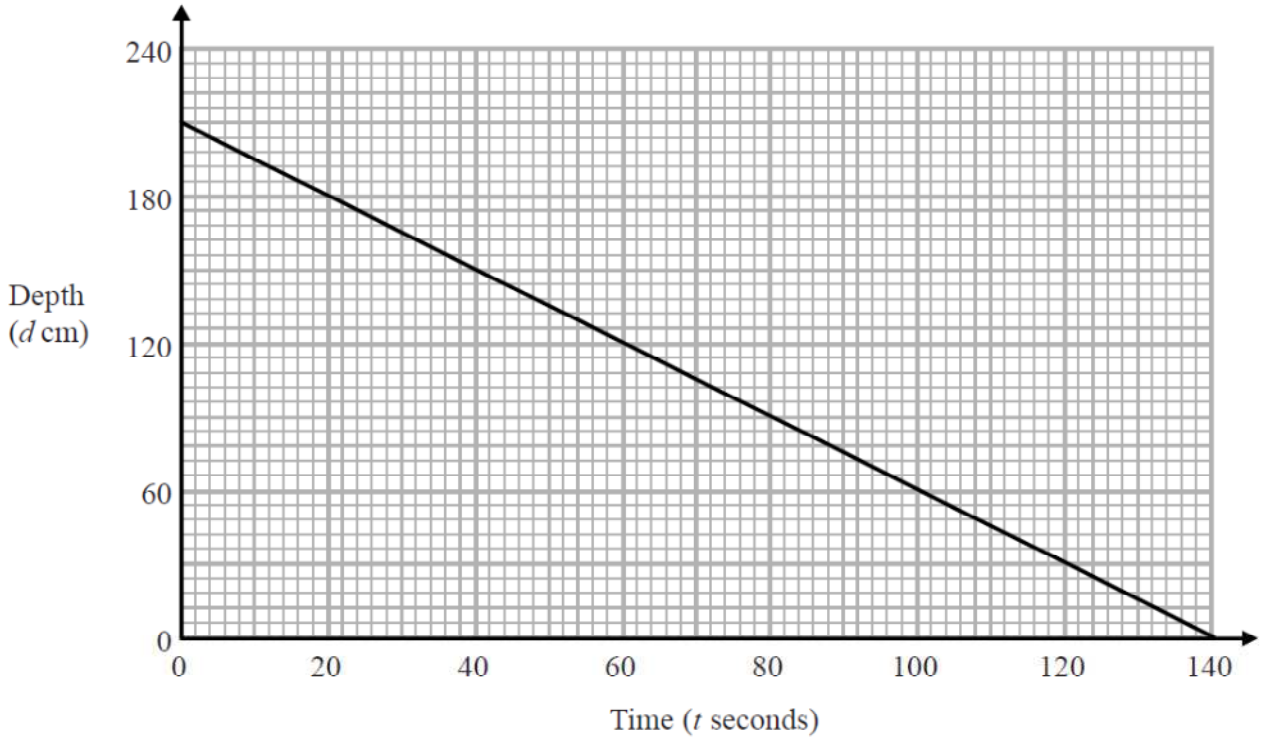
(a) Find the gradient of the straight line.
(b) Write down an interpretation of this gradient.

6. Joe heats some water in a kettle.
The graph gives information about the temperature of the water in the kettle and the length of time it has been heated.



(a) Find the gradient of the straight line.
(b) Write down an interpretation of this gradient.

7. The graph shows the depth, d cm, of water in a tank after t seconds.



(a) Find the gradient of this graph.
(b) Explain what this gradient represents.

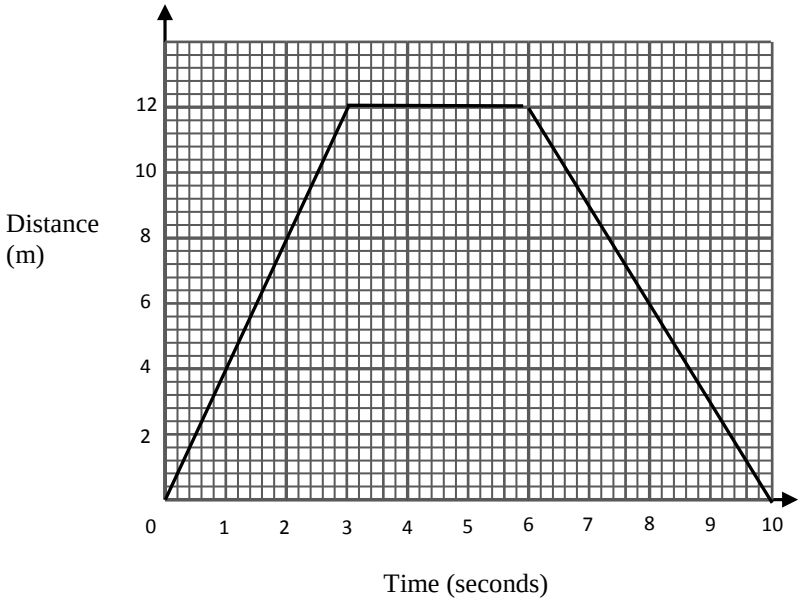
VELOCITY AND ACCELERATION

A distance-time graph shows the RATE OF CHANGE of distance with time.
It shows how an objects distance changes with time.

The GRADIENT of a distance-time graph tells you the **velocity** or **speed**.
The most common units of speed are ms^{-1}.

EXAMPLE 3

The distance-time graph shows the journey of a toy car.



(a)(i) Work out the gradient for the first 3 seconds. (ii) Interpret the gradient.
(b)(i) State the gradient between 3 and 6 seconds. (ii) Interpret the gradient.
(c)(i) Work out the gradient for the last 4 seconds. (ii) Interpret the gradient.

(a)(i) gradient $= \frac{\text{change in distance}}{\text{change in time}} = \frac{12}{3} = 4$ ← Use the line from 0 seconds to 3 seconds

(ii) The toy car is moving at a velocity of 4 ms^{-1} ← 12 metres ÷ 3 seconds = 4 metres per second

(b)(i) gradient = 0 ← The line has no slope. It is horizontal.

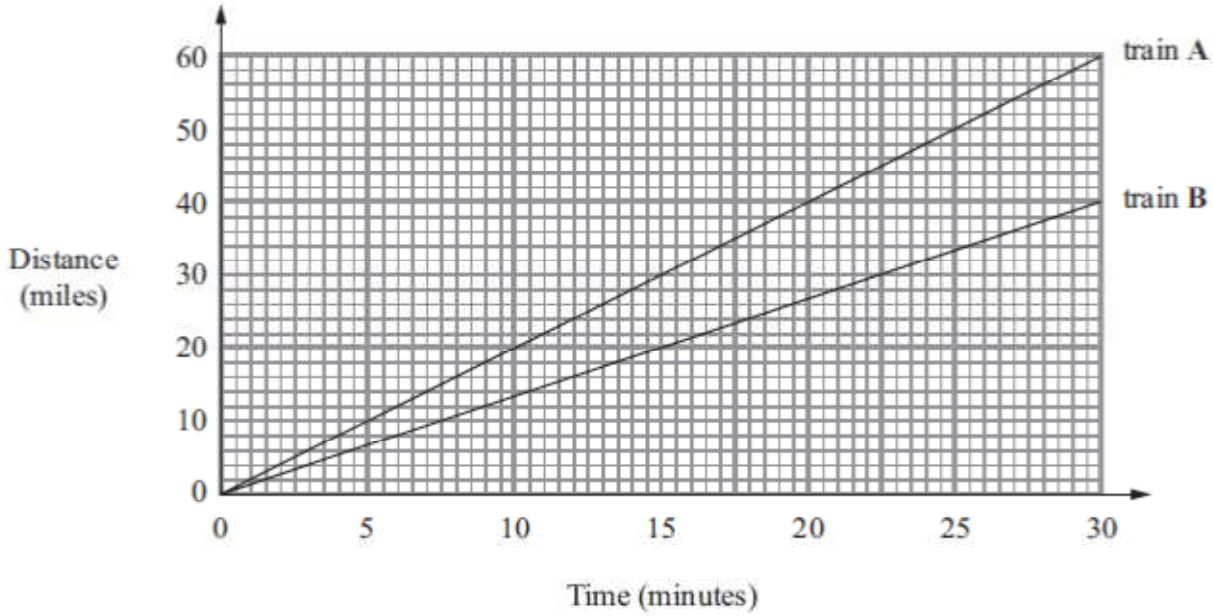
(ii) The car is stationary (not moving) ← The distance has not changed

(c)(i) gradient $= \frac{\text{change in distance}}{\text{change in time}} = -\frac{12}{4} = -3$ ← Negative gradient as line slopes down

(ii) The toy car is moving with a velocity of 3 ms^{-1}.
The negative sign shows the car is coming back to the start.

EXERCISE 2:

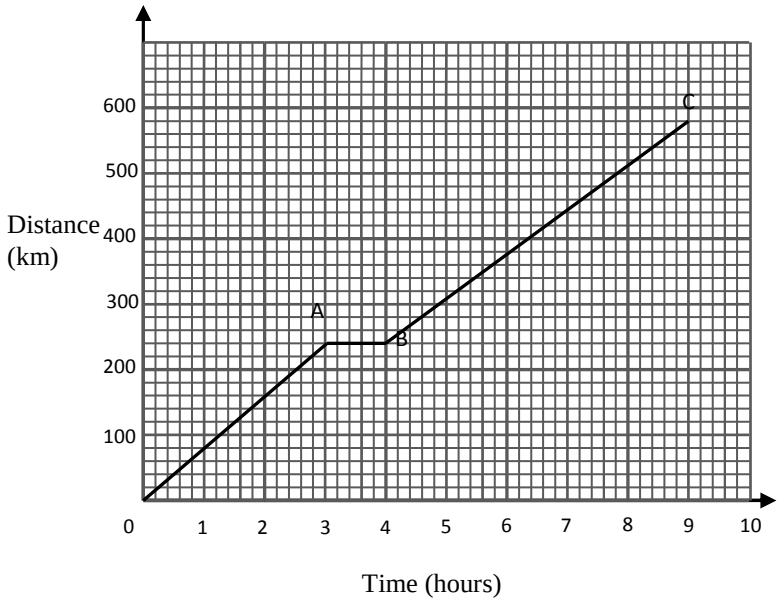
1. The graph shows the distance travelled by two trains.



(a) Work out the gradient of the line for train **A**.
(b) Interpret the gradient.
(c) Which train is travelling at the greater speed?

You must explain your answer.

2. The distance-time graph shows the journey of a train.



(a)(i) Work out the gradient for OA. (ii) Interpret this value
(b) Work out the speed for BC.
(c) Which part of the journey is the train travelling at a greater speed?

You must explain your answer.

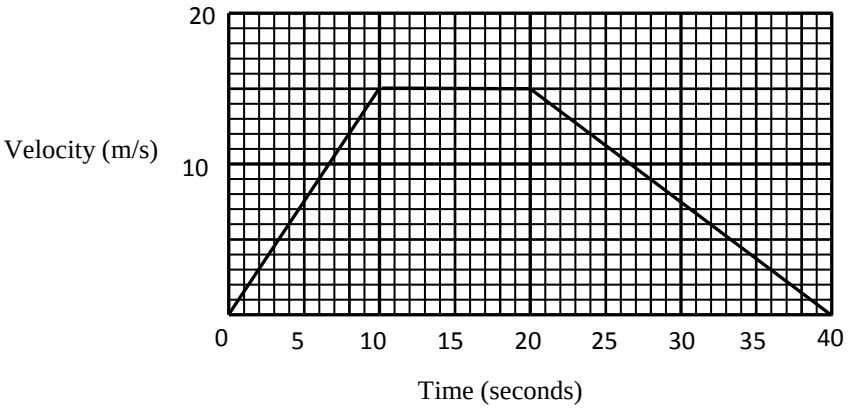
A velocity-time graph is sometimes called a speed-time graph.
Velocity means speed in a certain direction.

A velocity-time graph shows the RATE OF CHANGE of velocity with time.

The GRADIENT of a velocity-time graph tells you the **acceleration**.
The most common units of acceleration are ms^{-2}.

EXAMPLE 5

The velocity-time graph shows the velocity of a car for the first 40 seconds.



(a)(i) Work out the gradient for the first 10 seconds. (ii) Interpret the gradient.
(b)(i) State the gradient between 10 and 20 seconds. (ii) Interpret the gradient.
(c)(i) Work out the gradient for the last part of the journey. (ii) Interpret the gradient.

(a)(i) gradient $= \frac{\text{change in velocity}}{\text{change in time}} = \frac{15}{10} = 1.5$ ← Use the line from 0 seconds to 10 seconds

(ii) The training is moving with an acceleration of 1.5 ms^{-2}. It is speeding up. ← $15ms^{-1} \div 10\ s = 1.5\ ms^{-2}$

(b)(i) gradient = 0 ← The line has no slope. It is horizontal.

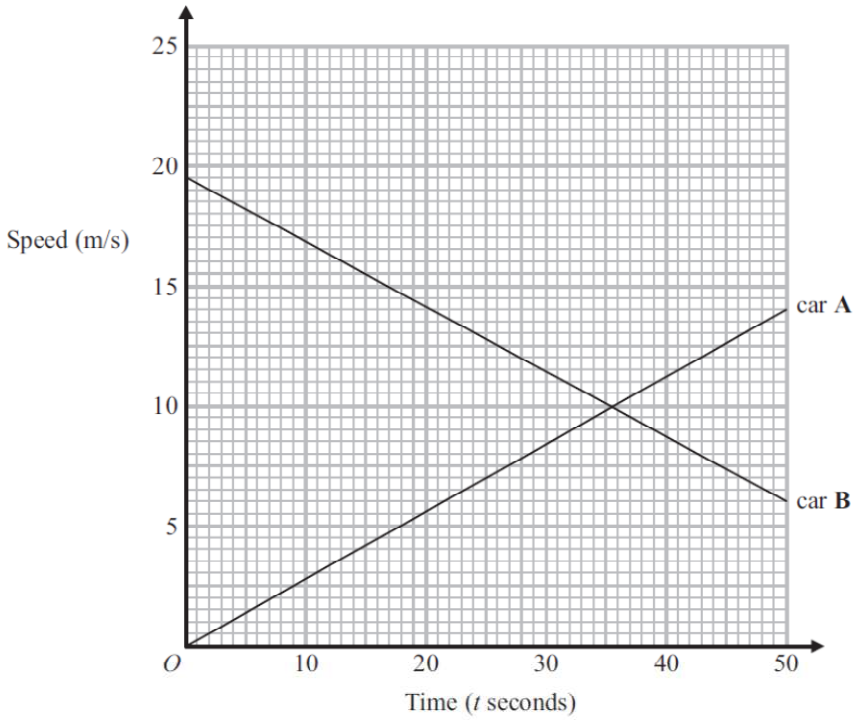
(ii) The train is travelling at a constant velocity. ← The velocity has not changed

(c)(i) gradient $= \frac{\text{change in velocity}}{\text{change in time}} = -\frac{15}{20} = -0.75$ ← Negative gradient as line slopes down

(ii) The train is moving with a deceleration of 0.75 ms^{-2}
It is slowing down.

EXERCISE 3:

1. The graph shows information about the speeds of two cars.



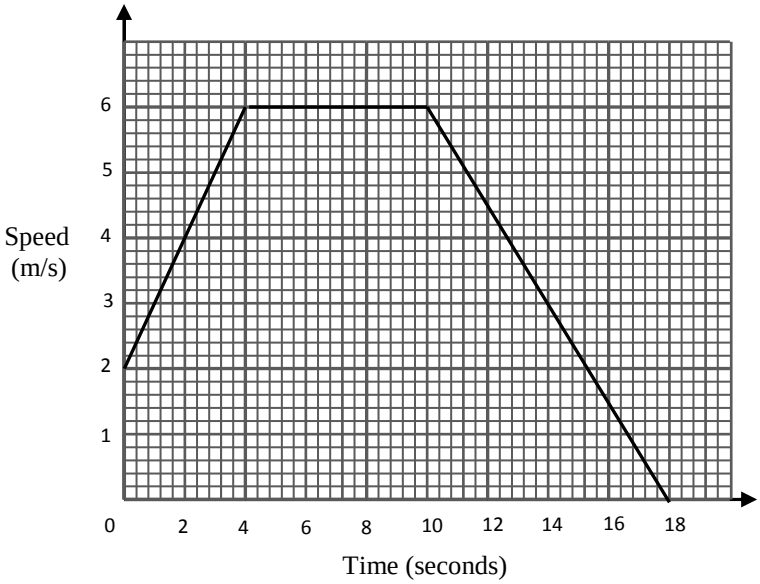
(a)(i) Work out the gradient of the line for car A. (ii) Interpret this value

(b) Work out the acceleration for car B.

(c) Which car, A or B, is slowing down?

You must explain your answer.

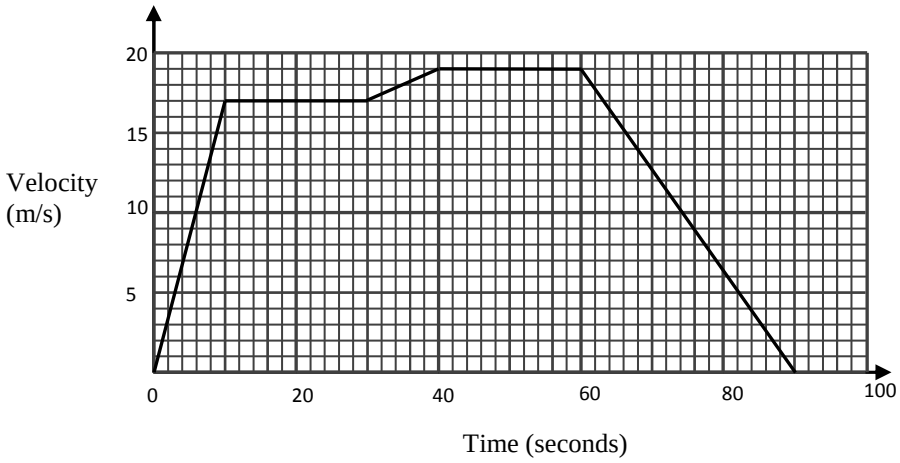
2. The diagram shows the speed-time graph for the last 18 seconds of Sandeep's cycle journey.



(a) Work out the acceleration.

(b) Work out the deceleration.

3. The graph shows the velocity of a motorbike over a period of 1.5 minutes.



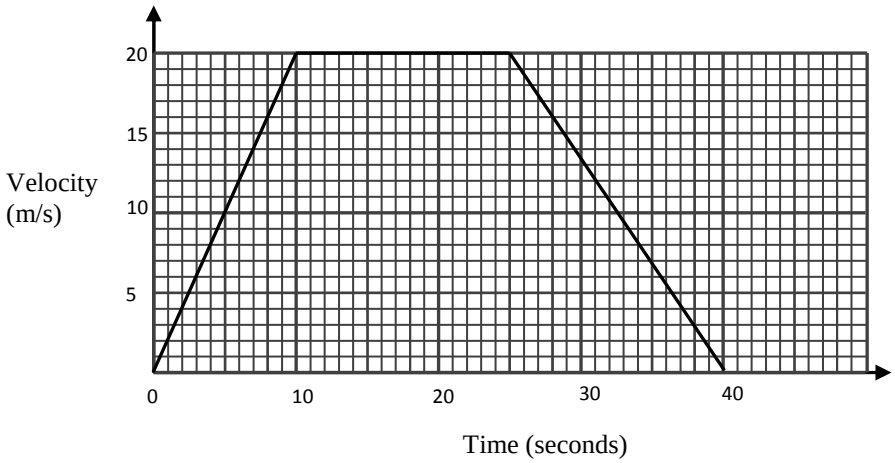
(a) Describe the motion of the motorbike over this period of 1.5 minutes.

(b)(i) During which time period is the acceleration the greatest?

You must explain your answer.

(ii) Work out this acceleration.

4. This graph shows a journey of a car.



(a) For how long is the car at a constant speed?

(b) What is the acceleration of the car?

(c) What is the deceleration of the car?

ANSWERS

Revision Exercise

1. (a) 2 (b) $\frac{4}{5}$ (c) 3 (d) $-\frac{1}{3}$ (e) −1

   (f) 0 (g) $\frac{1}{2}$ (h) 4 (i) $\frac{3}{2}$ (j) $-\frac{4}{5}$

2. $\frac{1}{2}$

3. −2

4. 3

5. (a) $\frac{3}{8}$ (b) $\frac{9}{5}$

Exercise 1

1. (a) £14
   (b) 1.2
   (c) For every cubic metre of water used the cost increases by £1.20
2. (a)(i) −0.35
   (ii) 0.35 litres of water is leaking per second
   (b)(i) −0.2
   (ii) 0.2 litres of water is leaking per second
   (c) P as its gradient is steeper
3. (a) 0.22
   (b) This means that 1 litre is equivalent to 0.22 gallons.
4. (a) −4
   (b) The depth of the water decreases by 4 cm per hour
5. (a) 40
   (b) The car travels a distance of 40 miles per gallon
6. (a) 2
   (b) The temperature increases by 2°C per second
7. (a) 1.5
   (b) The depth of the tank decreases by 1.5 cm per second

Exercise 2

1. (a) 2

   (b) Speed of train is 2 m/s

   (c) Train A is travelling at a greater speed as its gradient is steeper than the gradient of train B

2. (a)(i) 80 (ii) Speed of train is 80 km/h

   (b) 68

   (c) Journey OA is travelling at 80 km/h

   Journey BC is travelling at 68 km/h

   Hence, journey OA has the greater speed

Exercise 3

1. (a)(i) 0.28 (ii) This is the acceleration of car A which is 0.28 m/s^2

   (b) $-0.27\ m/s^2$

   (c) Car B is slowing down as its acceleration is negative

2. (a) $1\ m/s^2$

   (b) $-0.75\ m/s^2$

3. (a) 0 to 10 seconds: accelerating

   10 to 30 seconds: travels at constant speed

   30 to 40 seconds: accelerates

   40 to 60 seconds: travels at constant speed

   60 to 90 seconds: decelerates and then stops

   (b)(i) The first 10 seconds as the gradient is the steepest. (ii) $1.7\ m/s^2$

4. (a) 15 seconds

   (b) $2\ m/s^2$

   (c) $\frac{4}{3}\ m/s^2$